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QUANTUM RELATIVISTIC EQUATION AND STRING MASS QUANTIZATION

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ABSTRACT

New special relativistic energy relation is obtained by using the formal definition of force. This expression includes mass energy beside potential energy, with energy conserved. The effect of friction on energy lost is found by using uncertainty relation. The special relativistic energy in the presence of friction is found. This relation is used to find new special relativistic quantum equation. Treating particles as vibrating string the mass is quantized.

KEYWORDS:.

INTRODUCTION

Atomic and element are described by quantum mechanical laws[1,2]. Quantum laws succeeded in describing the atomic spectra and electronic structure of atoms[3], and the behaviour of single isolated atom. However, the description of behaviour of the bulk matter, which consist of a large number of interacting atoms, suffers from noticeable setbacks [4].For example the behaviour of high temperature super conductors (SC) cannot be described within the framework of ordinary quantum mechanics [5]. This may be related to the fact that SC are characterized by zero resistance and absence of friction[6].Quantum Laws are also unable to quantize gravitational field which described by general relativity [7]. This means that there is a need for relativistic quantum theory recognizing the frictional medium, thus it can hopefully quantize gravity and describe high temperature SC.
This task is done in sections (2) and (3). Sections (4) and (5) are devoted for discussion and conclusion.

RELATIVISTIC QUANTUM FRICTIONAL EQUATION:

If a particle move in frictional medium, its velocity and energy are lowered. This is since friction force apposes motion. Thus energy is dissipated to overcome friction effect. Relaxation time τ can be found from uncertainty principle by using the relation:

$$\Delta E \Delta \tau = \hbar \quad (1)$$

It is well known in laser physics that when an election is excited from ground state E_1 to an exited state E_2. The electron takes time τ in an excited state before returning back to the ground state. The relaxation time τ can be found from equation (1) to be:

$$\tau = \Delta t = \frac{\hbar}{\Delta E} \quad (2)$$

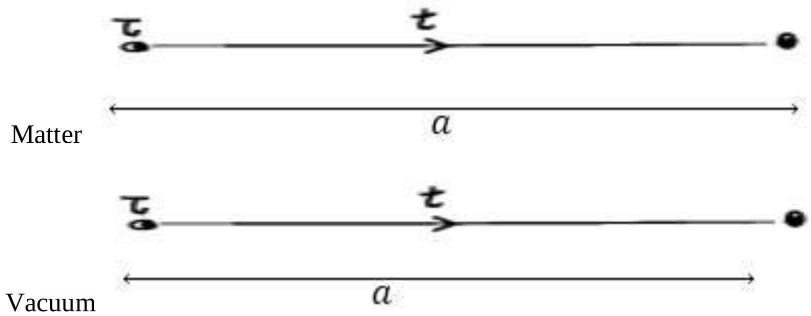
Where:

$$\Delta E = E_2 - E_1 \quad (3)$$

Therefore the energy loss by the electron when it leaves E_2 to E_1 is given by:

$$\Delta E = \frac{\hbar}{\tau} \quad (4)$$

This relation can be used to describe the lowering of the photon or light speed inside matter. When a photon is incident on a certain atom, it can be absorbed by it to make an electron leave $E_1$to an excited state $E_2$by emitting a photon. It return back to E_1.This means that the photon, instead of taking time t moving with speed c:



in vacuum, it take a time $t+\tau$ inside matter, with delay timeτ. This means that as if light moves with apparent speed v inside matter where:

$$v=\frac{a}{(t+\tau)}$$

$$a=v(t+\tau) \quad (5)$$

While it moves the same distance in vacuum with speed c, such that:

$$a=ct \quad (6)$$

From (5) and (6):

$$a=v\left(\frac{a}{c}+\tau\right)=\frac{v}{c}[a+c\tau]$$

Thus the refractive index n is given by:

$$n=\frac{c}{v}=\left[1+\frac{c\tau}{a}\right]$$

Therefor the relaxation time is given by:

$$\tau=(n-1)\frac{a}{c} \quad (7)$$

Where a here stands for the distance between neigbouring atoms. This delay time thus is responsible for lowering the light speed from $c\ to\ v$.

this is equivalent to existence of friction that causes energy loss given by equation (4) to be:

$$\Delta E=\frac{\hbar}{\tau} \quad (8)$$

This means that when a particle of original energy E_0 enters a frictional medium its energy is lowered to be come

$$E=E_0-i\Delta E=E_0-\frac{i\hbar}{\tau} \quad (9)$$

Where one uses the complex representation according to the special relativistic theory the original energy is given by:

$$E_0=mc^2 \quad (10)$$

But if one defines the force to be:

$$F=\int\frac{dmv}{dt}.d\underline{\tau}=-\int\nabla V.d\underline{V} \quad (11)$$

Where V stands for potential energy:

$$\int\underline{v}.dmv=-\int dV \quad (12)$$

Using the relation:

$$m=\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \quad (13)$$

One can easily find that:

$$mc^2=-V+C \quad (14)$$

$$mc^2 + V = C = Constant \quad (15)$$

This constant of motion is assumed to represent the energy E_0 where:

$$E_0 = mc^2 + V = \frac{m_0 mc^4}{\sqrt{m^2c^4 - m^2v^2c^2}} + V$$

$$(E_0 - \mathrm{V})^2(m^2c^4 - P^2c^2) = m_0{}^2m^2c^4$$

$$\cancel{m^2c^4}(m^2c^4 - P^2c^2) = m_0{}^2c^4\cancel{m^2c^4}$$

$$m^2c^4 = P^2c^2 + m_0{}^2c^4$$

$$(E_0 - \mathrm{V})^2 = P^2c^2 + m_0{}^2c^4 \quad (16)$$

For very small rest mass energy, one can write:

$$E_0 = cP + V \qquad (17)$$

In view of equation (9) the energy in the pressure of friction:

$$E = E_0 - \frac{i\hbar}{\tau} = cP + V - \frac{i\hbar}{\tau} \quad (18)$$

To find quantum equation for this expression of relativistic energy for frictional medium, one multiply (18) by the wave:

$$E\Psi = cP\Psi - \frac{i\hbar}{\tau}\Psi + V\Psi \qquad (19)$$

Using the wave function for quantum system:

$$\Psi = Ae^{\frac{i}{\hbar}(Px - Et)}$$

One can obtain:

$$i\hbar\frac{\partial\Psi}{\partial t} = E\Psi \qquad \frac{\hbar}{i}\nabla\Psi = P\Psi \quad (20)$$

Thus inserting equation (20) in equation (19) yields:

$$i\hbar\frac{\partial\Psi}{\partial t} = c\frac{\hbar}{i}\nabla\Psi + V\Psi - \frac{i\hbar}{\tau}\Psi \qquad (21)$$

Another two new equations can also be obtained from (16) to get:

$$E_0{}^2 - 2VE_0 + V^2 = P^2c^2 + m_0{}^2c^4 \quad (22)$$

The friction effect can be replacing E_0 by E, where:

$$E = E_0 - \frac{i\hbar}{\tau} \qquad (23)$$

Thus equation (22) reads:

$$E^2 - 2VE + V^2 = P^2c^2 + m_0{}^2c^4$$

$$V^2 + \left(E_0 - \frac{i\hbar}{\tau}\right)^2 - 2V\left(E_0 - \frac{i\hbar}{\tau}\right) = P^2c^2 + m_0{}^2c^4$$

$$V^2 + E_0{}^2 + \frac{\hbar^2}{\tau^2} - 2\frac{i\hbar}{\tau}E_0 - 2VE_0 + 2\frac{i\hbar}{\tau}V = P^2c^2 + m_0{}^2c^4 \qquad (24)$$

Multiplying both sides by Ψ Yields:

$$E_0{}^2\Psi + V^2\Psi + \frac{\hbar^2}{\tau^2}\Psi - 2\frac{i\hbar}{\tau}E_0\Psi - 2VE_0\Psi + 2\frac{i\hbar}{\tau}V\Psi = P^2c^2\Psi + m_0{}^2c^4\Psi \qquad (25)$$

Using the wave-particle dual nature relation:

$$\Psi = Ae^{\frac{i}{\hbar}(Px - E_0t)}$$

Yields:

$$-\hbar^2\frac{\partial^2\Psi}{\partial \mathrm{t}^2} = E_0{}^2\Psi \quad , \quad i\hbar\frac{\partial\Psi}{\partial t} = E_0\Psi$$

$$-\hbar^2\nabla^2\Psi = P^2\Psi \qquad (26)$$

Inserting (26) in (25) results in the following equation:

$$-\hbar^2\frac{\partial^2\Psi}{\partial \mathrm{t}^2} + V^2\Psi + \frac{\hbar^2}{\tau^2}\Psi + 2\frac{\hbar^2}{\tau}\frac{\partial\Psi}{\partial t} - 2i\hbar V\frac{\partial\Psi}{\partial t} + \frac{2i\hbar}{\tau}V\Psi = -c^2\hbar^2\nabla^2\Psi + m_0{}^2c^4\Psi$$

Which is the relativistic equation in the presence of friction. In the absence of friction equation (26) reduces to:

$$-\hbar^2\frac{\partial^2\Psi}{\partial \mathrm{t}^2} - 2i\hbar v\frac{\partial\Psi}{\partial t} + V^2\Psi = -c^2\hbar^2\nabla^2\Psi + m_0{}^2c^4\Psi \quad (27)$$

Harmonic oscillator solution:

To solve (27) for to simplify the oscillator, it is suitable to simplify the equation by suggesting:

$$\Psi = ue^{-\frac{i}{\hbar}Et} \quad (28)$$

To get:

$$E^2u - 2EVu + V^2u = -c^2\hbar^2\nabla^2\mathrm{u} + {m_0}^2c^4\mathrm{u} \quad (29)$$

For harmonic oscillator.

$$V = \frac{1}{2}kx^2$$

Therefore x is small thus one can neglect terms like V^2 to get:

$$E^2u - Ekx^2u = -c^2\hbar^2\nabla^2\mathrm{u} + {m_0}^2c^4\mathrm{u} \quad (30)$$

Since:

$$k = m\omega^2 \quad (31)$$

$$E^2u - m\omega^2Ex^2u = -c^2\hbar^2\nabla^2\mathrm{u} + {m_0}^2c^4\mathrm{u} \quad (32)$$

Consider now a solution:

$u = u_0e^{-\alpha x^2}$ $\quad\quad$ $\nabla u = -2\alpha xu$

$\nabla^2u = -2\alpha u + 4\alpha^2x^2u$ $\quad (33)$

Substituting (33) in (32) yields:

$[E^2 - m\omega^2Ex^2]u = [2\alpha c^2\hbar^2 - 4\alpha^2c^2\hbar^2x^2]\mathrm{u} + {m_0}^2c^4\mathrm{u} \quad (34)$

Comparing the free terms and coefficients of x^2 on both sides yields:

$$2\alpha c^2\hbar^2 = E^2 - {m_0}^2c^4 \quad (35)$$

$$4\alpha^2c^2\hbar^2 = m\omega^2E \quad (36)$$

Dividing (36) by (35) after ignoring the rest mass yields:

$$2\alpha = \frac{m\omega^2}{E} \quad (37)$$

To find E, by ignoring m_0 for very small mass:

$$4\alpha^2c^4\hbar^4 = E^4$$

$$\frac{E^4}{c^2\hbar^2} = m\omega^2E$$

$$E^3 = c^2\hbar^2m\omega^2 \quad (38)$$

But according to Einstein equation:

$$mc^2 = E = \hbar\omega \quad (39)$$

Thus:

$$E^3 = \hbar^3\omega^3$$

Hence:

$$E = \hbar\omega \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (40)$$

Another solution can be proposed by using periodicity condition of equation (28), where

$$\Psi(t+T) = \Psi(t) \quad (41)$$

Which requires:

$$\mathrm{e}^{-\frac{iE}{\hbar}T} = 1$$

$$\cos\theta = 1 \quad\quad \sin\theta = 0$$

$$\theta = \frac{E}{\hbar}T = 2n\pi \quad (42)$$

Thus:

$$E = \frac{2\pi}{T}\mathrm{n}\hbar = n\hbar\omega \quad (43)$$

Substituting in (38) yields:

$$m = n^3\frac{\hbar\omega}{c^2} \quad (44)$$

Thus the mass is quantized.

From (37), (43) and (44):

$$\propto = \frac{m\omega^2}{2n\hbar\omega} = \frac{m\omega}{2n\hbar} = \frac{n^3\hbar\omega^2}{2n\hbar c^2} = \frac{n^2\omega^2}{2c^2} \quad (45)$$

DISCUSSION

The energy lost by friction can be found by using uncertainty principle in section (2). Equation (4) shows that it is inversely proportional to relaxation time τ. This expression conforms with the classical one in which friction energy is inversely proportional to τ. This friction energy is added to the SR energy found by adding potential terms to secure energy conservation as shows in equation (15). By neglecting rest mass the energy is given by (17). The final SR term is given in equation (18) consisting of kinetic, potential and frictional terms. Using the wave equation for particles in equation (20) a relativistic quantum expressions for neglected rest mass and non-neglected m_0 were found as shown by equations (21) and (26). Neglecting friction, one finds equation (27) this equation is a modified SR equation. This equation (27) is used to solve for harmonic oscillator, within the frame work of string theory. The solution shows that the energy and mass are quantized, as shown by equation (43) and (44).

CONCLUSION

The new SR quantum based on energy expression including matter energy and potential energy is promising. It shows that the mass is quantized within the frame work of string theory.

REFERENCES

[1] Wald and Robert, quantum theory in carvel space-time , University of Chicago. 2011.
[2] Lavish, B. G., Theoretical Physics, John Wiley and sons, New York 2009
[3] Eppley, Kenneth and Eric, The necessity of quantizing the gravitational field, foundation of phy.2013.
[4] M. Green, J. Schwarz and E. Witten, “Superstring Theory”, volume I and II,Cambridge University Press, 2010.
[5] D. L¨ust and S. Theisen, “string theory”, , Springer Verlag, 2009.
[6] B. Zwiebach, “A first course in string theory”, Cambridge university press, 2004.
[7] J. Polchinski, “String theory”, volume I and II, Cambridge university press, 2009.
[8] Derek, F. Lowden, “An introduction to tensor calculus and relativity, John Wiley and sons, New York,2013.